10-6 Exponential Growth and Decay

Depreciation

$y = a(1-r)^t$

y = new amount
a = original amt.
% $r/k \rightarrow$ rate
t = time

Decay

$y = ae^{-kt}$

EX: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?

$y = a(1-r)^t$

$65 = 130(1-.11)^t$

$\frac{1}{2} = (.89)^t$

$\log \frac{1}{2} = \log .89^t$

$\log .5 = t \log .89$

$\frac{\log .5}{\log .89} = t$

$5.9 \text{hrs} = t$

How long will it take for 90% of this caffeine to be eliminated from a person's body?

$13 = 130(.89)^t$

$\frac{1}{10} = .89^t$

$19.8 \text{hrs} = t$

EX: The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated. All life on Earth contains the radioactive element Carbon-14, which decays continuously at a fixed rate. The half-life of Carbon-14 is 5760 years; that is, every 5760 years, half of a mass of Carbon-14 decays away.

What is the constant for Carbon-14?

$y = ae^{-kt}$

$\frac{1}{2} = 1e^{-k(5760)}$

$\frac{1}{2} = e^{-5760k}$

$\ln \frac{1}{2} = -5760k$

$.00012 = k$

A paleontologist examining the bones of a wooly mammoth estimates that they contain only 3% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

$$y = ae^{-.00012t}$$
$$.03 = 1e^{-.00012t}$$
$$\ln .03 = -.00012t$$
$$29{,}221\text{yrs} = t$$

EX: The half-life of Sodium-22 is 2.6 years. What is the constant for Sodium-22?

$$\frac{1}{2} = 1e^{-k(2.6)}$$
$$.2666 = k$$

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of the Earth?

8.6yrs

Appreciation

$y = a(1 + r)^{t}$

Growth

$y = ae^{kt}$

EX: In 1910 the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in the year 2010?

$y = 120{,}000(1.015)^{100}$

$y = 531{,}845$

$y = 120{,}000(1 + .015)^{100}$

HW
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